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MINIMUM TIME TRAJECTORY GENERATION FOR RELATIVE GUIDANCE OF AIRCRAFT

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ABSTRACT

In this communication is considered the problem of on line generation of minimum time trajectories to be followed by an aircraft to achieve relative convergence maneuvers.

Here the trajectory generation problem is first considered as a minimum time control problem. The analysis of the resulting set of complex optimality conditions shows that the minimum time trajectories are produced by bang-bang control laws and can be characterized by some few geometric parameters. Then *regular* minimum time convergence trajectories can be defined.

For practical considerations it appears necessary to solve on line this problem. Taking into consideration the structure of the regular minimum time convergence trajectories, an equivalent mathematical programming problem is formulated. .

An off line exhaustive solution approach, based on reverse dynamic programming, is proposed to cover a large set of initial relative positions. Then a feed forward neural network can be trained to associate to an initial relative position the parameters of an optimal regular minimum time convergence trajectory.

INTRODUCTION

In this communication is considered the problem of on line generation of minimum time trajectories to be followed by an aircraft to achieve relative convergence maneuvers. With the development of the *free flight* and *autonomous aircraft* concepts in Civil Aviation in the recent years, this problem has risen increased interest.

Considering the current and predicted levels of congestion of air traffic, studies related to the delegation to the flight crew of some tasks currently performed by air traffic controllers are actively tackled today [1]. Among these studies, relative guidance between aircraft has appeared to be promising for the increase of air traffic capacity. The objective of this communication is to provide a technical insight into the airborne devices and algorithms which may be used to automatically perform this new type of maneuver. From an operational point of view, and assuming normal operations, the air traffic controller is relieved of providing instructions to the trailing aircraft for merging behind the leading aircraft and maintaining a given spacing once the flight crew has accepted a relative guidance clearance. (see figure 1 for an example of traffic situation). Thus, the expected benefit of such new capabilities onboard aircraft is an increase of air traffic controller availability, which would result in increased air traffic capacity and/or safety. Enhancement of flight crew airborne traffic situational awareness is also expected with associated safety benefits. The feasibility of such a relative guidance device is based on the ability of each aircraft to broadcast and receive suitable navigation data thanks to Automatic Dependent Surveillance-Broadcast (ADS-B)[2]. Among those navigation data, identification, position, speed and heading are of interest for the design of minimum time convergence guidance control laws.

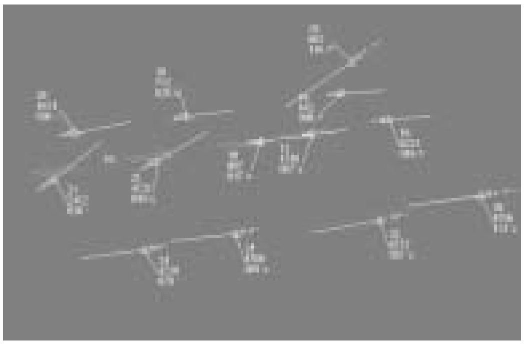

Figure 1: Example of radar image at Paris-Charles de Gaulle

Here the trajectory generation problem is first considered as a minimum time control problem and optimality conditions (Pontryagine's maximum principle [3]) are derived from the mathematical formulation of the control problem. The analysis of the corresponding optimality conditions shows that the minimum time trajectories resulting from optimal bang-bang control laws can be characterized by some few geometric parameters while *regular* minimum time convergence trajectories can be defined. Then a new mathematical programming problem can be formulated from the original optimal control problem.

Since the evolution of aircraft is in general subject to perturbations (winds, navigation and guidance errors, etc), it appears necessary to solve on line this mathematical programming problem according with the current situation. Considering the complexity of the problem and since aircraft dynamics are fast, the effective on line resolution of this problem is not feasible.

However a practical solution strategy, composed of two steps, is proposed in this communication:

First, taking advantage of the properties of the regular minimum time trajectories, an off line exhaustive solution approach, based on reverse dynamic programming, can be developed to cover a large set of initial relative positions.

Then a feed forward neural network is built and trained to associate to initial relative positions an optimal regular minimum time trajectory.

Finally, once the trained neural network is available, it can be solicited periodically to provide, either to the aircraft's auto pilot or to the air traffic controller, new updates for the minimum time trajectory parameters. When directed to the autopilot, this information is used to build reference values for the flight control laws, while when directed to the air traffic control officer; it provides minimum time bounds for the completion of a convergence maneuver.

The proposed communication is organized as follows: The trajectory generation problem is described and formulated, the optimality conditions are analyzed, the regular minimum time trajectories are introduced, the data generation through reverse dynamic programming is described as well as the feed forward neural networks and finally a numerical example is displayed.

PROBLEM FORMULATION

The case where a trailing aircraft is made to converge towards the trajectory of another aircraft declared to be the leader is considered in this study. The two aircraft are supposed to maintain their common flight level and their respective speeds until the convergence maneuver is completed. This maneuver is considered to be completed once the trailing aircraft satisfies the following conditions: its speed is parallel to the leader's speed, it follows the same track and its separation from the leader is larger than the minimum separation imposed by the rules of Civil Aviation (distance $D \geq d_{min}$). The equations describing the relative motion of the two aircraft are given by:

$$\dot{d} = V_L \cos(\theta - \psi_L) - V_S \cos(\theta - \psi_S) \quad (1)$$

$$\dot{\theta} = \{-V_L \sin(\theta - \psi_L) + V_S \sin(\theta - \psi_S)\}/d \quad (2)$$

$$\dot{\psi}_S = r_S \quad (3)$$

where V_L and V_S are respectively the speed of the leading aircraft and the speed of the trailing aircraft, d(t) is the instant separation between the two aircraft, ψ_L et ψ_S are their respective headings and θ is the angle of sight with respect to the reference direction (see figure 2).

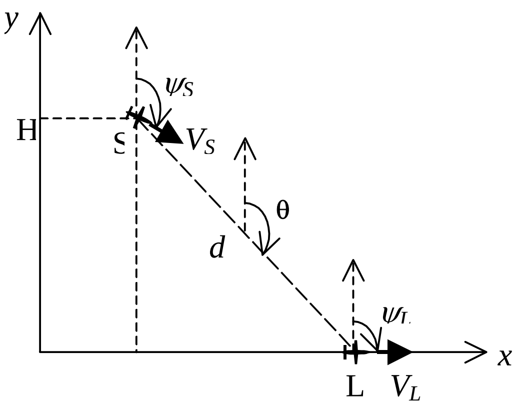

Figure 2 : Relative position of aircraft

A measure of the effectiveness of such a maneuver can be given by its duration. A minimum time convergence problem whose solution will provide a feasible convergence trajectory for the trailing aircraft is then considered. This problem can be formulated as:

$$\min \int_0^{t_f} dt \qquad (4)$$

under the dynamical constraints (1), (2) and (3) and the limit conditions:

$$\phi_{\min} \le \phi_S \le \phi_{\max} \text{ and } d_{\min} - d \le 0 \qquad (5)$$

where ϕ_S is the bank angle of the trailing aircraft. Initial conditions must be met:

$$d(0) = d_0 \text{ with } d_0 \ge d_{\min}\,,$$
$$\theta(0) = \theta_0\,, \quad \psi_S(0) = \psi_{S0} \qquad (6)$$

as well as final convergence conditions :

$$\psi_S(t_f) = \psi_L\,,\ \ \theta(t_f) = \psi_L\,,\ \ d(t_f) = D \qquad (7)$$

The independent variable here is time t with final value t_f. The control variable appears here to be the yaw rate r_s of the trailing aircraft, but since it is supposed that all turn maneuvers are performed in stable conditions, the following relation holds:

$$r_S = (g / V) \cdot tg\phi_s \qquad (8)$$

and the bank angle of the trailing aircraft ϕ_s can be taken ultimately as the actual control variable which must be set by the lateral function of the Auto Pilot of the aircraft. It is also worth to observe that the separation at final convergence D, must be chosen within a feasible range by the pilot of the trailing aircraft of by a traffic controller in charge of this sector.

OPTIMALITY CONDITIONS

Making use of classical results of optimal control theory [3], an Hamiltonian H is associated to the above minimum time optimization problem:

$$H(\theta, d, \psi_S, \lambda_\theta, \lambda_d, \lambda_\psi, \mu, \nu, r_S) =$$
$$1 + \lambda_d \{V_L \cos(\theta - \psi_L) - V_S \cos(\theta - \psi_S)\}$$
$$+ \lambda_\theta \{-V_L \sin(\theta - \psi_L) + V_S \sin(\theta - \psi_S)\}/d$$
$$+ \lambda_\psi r_S + \mu\left(d_{\min} - d + \nu^2\right) \qquad (8)$$

where λ_d, λ_θ and λ_ψ are the co-state variables associated to the differential equations (1), (2) and (3) which provide the rates of change of the state variables d, θ et ψ. When considering the minimum separation constraint, a dual variable μ is introduced as well as a slack variable ν, which should be such as:

$$\mu(t) \ge 0 \quad \text{and} \quad \mu(t)\ \nu(t) = 0 \qquad (9)$$

The optimality conditions can then be established. They are composed of the following terms:
1) Hamilton's canonical equations:

$$d\underline{X}/dt = \partial H/\partial \underline{\lambda} \quad \text{and} \quad d\underline{\lambda}/dt = -\,\partial H/\partial \underline{X} \qquad (10)$$

with $\underline{X}' = (d, \theta, \psi)$ and $\underline{\lambda}' = (\lambda_d, \lambda_\theta, \lambda_\psi)$

2) the transversality conditions dealing here with initial and final conditions for the state vector (relations (6) et (7)).
3) the minimum principle which states that "the optimal solution is the one which minimizes the Hamiltonian while meeting the constraints".

Then to find the solution of this minimum time convergence problem, a sixth order non-linear two-point boundary problem should in theory be solved. The direct resolution of such problem is in numerical grounds very complex [3, 4, 5] and does not suit with a real time airborne implementation, especially when civil aviation aircraft are concerned.

However, the analysis of the minimization conditions of the associated Hamiltonian with respect to the control variable r_s provides some insight over the optimal trajectory and related operational conditions. Indeed, the above Hamiltonian presents an affine form with respect to the control variable since it can be written such:

$$H = f(\theta, d, \psi_S, \lambda_\theta, \lambda_d, \mu, \nu) + \lambda_\psi r_S \qquad (11)$$

Then its minimization with respect to the control variable leads to a solution of the "bang bang" type (here the attached asterisk indicates that the values are those of an optimal solution):

$$r_S^*(t) = g\ tg(\phi_{\max}) / V_S \qquad \text{if } \lambda_\psi^*(t) < 0 \qquad (12.1)$$

$$r_S = 0 \text{ and } \phi_S = 0 \text{ if } \lambda_\psi^*(t) = 0 \qquad (12.2)$$

and

$$r_S^*(t) = -g\ tg(\phi_{\max}) / V_S \text{ if } \lambda_\psi^*(t) > 0 \qquad (12.3)$$

This type of solution has been studied in detail in [7]. It can be shown that the corresponding optimal trajectory is composed of maximum left or right bank angle turns according to the sign of the co-state variable $\lambda_\psi^*(t)$, and by straight evolutions at zero bank angle when this variable maintains the zero value.

CHARACTERIZATION OF MINIMUM TIME CONVERGENCE TRAJECTORIES

The analysis of the optimality conditions shows that the minimum time convergence trajectories are composed of maximum bank angle turns (right or left) and of linear segments. The number and the duration of these components depend of the initial relative position of the aircraft as well as of their respective speeds. Three main cases can be then considered:
- one where "direct" convergence is possible. In this case the optimal convergence trajectory is composed of a straight-line segment followed by a final turn maneuver to join the route of the leading aircraft.
- one where an initial turn is necessary to insure a converging track for the trailing aircraft towards the route of the followed aircraft. In this case, the minimum time convergence trajectory is composed of two opposite maximum bank angle urns separated by a straight-line segment.
- one where a preliminary separation maneuver is necessary before starting convergence. In this ase the convergence trajectory will be composed of three maximum bank angle turns separated by two straight line segments (one of them can be reduced to an inflexion point).

A *regular convergence trajectory of order n* (see figure 3 for examples of different orders) can then be defined as a trajectory composed of a succession of (n+1) pairs, each pair being composed of a straight line segment and a maximum bank angle turn.

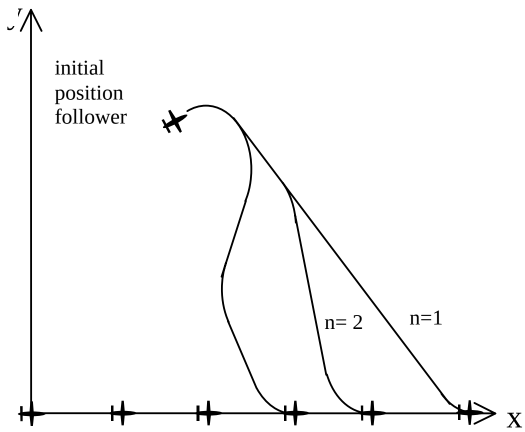

Figure 3 : Examples of regular convergence trajectories with different orders.

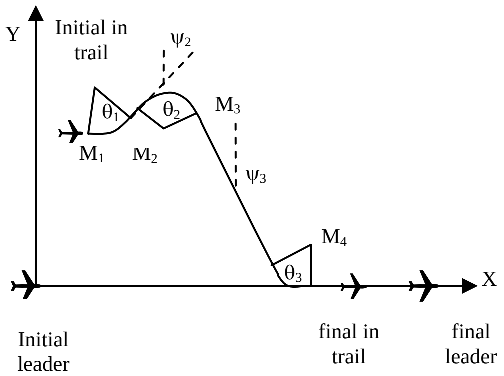

Figure 4 : Representation of a regular convergence trajectory.

The regular convergence trajectory displayed in figure 4 is characterized by a succession of triplets (ε_i, θ_i, l_i).
Here ε_i is a binary variable giving the orientation of the i^{th} maximum bank angle turn ($\varepsilon_i = +1$: left turn, $\varepsilon_i = -1$: right turn), θ_i is the absolute angular value of the i^{th} turn and L_i is the length of the

straight line segment leading to the i[th] maximum bank angle turn.

CHOICE OF A REGULAR CONVERGENCE TRAJECTORY

Let $P_n((l_i,\ \varepsilon_i,\ \theta_i)_{i\ =\ 1\ à\ n-1})$ be the problem of definition of a minimum time regular convergence trajectory which starting from the initial conditions satisfies as well the convergence constraints as the minimum separation constraints.

Let $S_n^* = \left(l_i^n, \varepsilon_i^n, \theta_i^n\right)_{i=1\,à\,n-1}$ be the solution of this problem. From this solution it is possible to build trivially feasible solutions of higher orders. For instance, for P_{n+1}, a feasible solution $\tilde{S}_{n+1}$ is given by:

$$\tilde{S}_{n+1} = \left(l_i^n, \varepsilon_i^n, \theta_i^n\right)_{i=1\,à\,n-1} \oplus \left(l_n^n, \varepsilon_n^n, \theta_n^n\right)$$

$$\text{with } l_n^n = 0,\ \theta_n^n = 0 \text{ and } \varepsilon_n^n = \pm 1 \tag{13}$$

$\oplus$ is here a concatenation operator for chains of the above triplets.

These solutions having the same duration, it appears that problem P_{n+1} will have a performance at least equal to the one of problem Pn. This result can appear to be paradoxical but it is useful to observe that the final convergence point is not fixed and that minimum separation constraints can become active. However, in the case in which the final convergence point is fixed, the solution of problem P_n remains interesting since it allows to know if the convergence at this given point is feasible with a n[th] order trajectory.

It is desirable to limit as much as possible the number of elementary maneuvers to perform overall minimum time convergence maneuvers so that the comfort of passengers and crews remains at an acceptable level and the workload of air traffic controllers does not become excessive. Then, the proposed trajectory generation procedure adopts the following steps:

- first chose a maximum order for candidate regular convergence trajectories: n_{max} ,
- then solve problem $P_{n_{\max}}$.
- if this problem has no feasible solution the intended maneuver will be deleted.

CONSTRUCTION OF MINIMUM TIME REGULAR TRAJECTORIES

Considering the solution of the optimization P_n figure 2 shows that the duration of a regular convergence maneuver (from point M_n to point M_1) is given by:

$$t_n = \sum_{k=1}^{n-1}\left(l_k + R_{\min}\theta_k\right)/V_S \tag{14}$$

The convergence conditions can be rewritten:

$$x_1 = (V_L / V_S)\sum_{k=1}^{n-1}\left(l_k + R_{\min}\theta_k\right) - D \quad , \quad y_1 = 0, \quad \psi_1 = \psi_L$$

$$\text{with} \quad y_1 = 0 \text{ and } \psi_1 = \psi_L \tag{15-1}$$

Starting from the convergence point M_1 with coordinates x_1, y_1 (in a reference frame tangent to the initial trajectory of the leading aircraft) and with a heading ψ_1 equal to ψ_L, the leader's initial heading, it is possible to get successively the coordinates of each of the characteristic points of the regular convergence trajectory as a function of the parameters l_i, ε_i et θ_i, i = 1 à n-1 .

One gets for point M_2 :

$$\begin{cases} x_2 = x_1 + \varepsilon_1 R_{\min}\left(\cos\left(\psi_1 + \varepsilon_1\theta_1\right) - \cos\psi_1\right) \\ -\, l_1 \sin\left(\psi_1 + \varepsilon_1\theta_1\right) \\ y_2 = y_1 + \varepsilon_1 R_{\min}\left(\sin\psi_1 - \sin\left(\psi_1 + \varepsilon_1\theta_1\right)\right) \\ -\, l_1 \cos\left(\psi_1 + \varepsilon_1\theta_1\right) \\ \psi_2 = \psi_1 + \varepsilon_1\theta_1 \end{cases} \tag{15-2}$$

and finally for point M_n :

$$\begin{cases} x_2 = x_1 + \varepsilon_1 R_{\min}\left(\cos\left(\psi_1 + \varepsilon_1\theta_1\right) - \cos\psi_1\right) \\ -\, l_1 \sin\left(\psi_1 + \varepsilon_1\theta_1\right) \\ y_2 = y_1 + \varepsilon_1 R_{\min}\left(\sin\psi_1 - \sin\left(\psi_1 + \varepsilon_1\theta_1\right)\right) \\ -\, l_1 \cos\left(\psi_1 + \varepsilon_1\theta_1\right) \\ \psi_2 = \psi_1 + \varepsilon_1\theta_1 \end{cases} \tag{15-n}$$

The initial conditions become:

$$x_n = x_0^S,\ y_n = y_0^S \text{ and } \psi_n = \psi_0^S \tag{16}$$

The minimum separation constraints take the form:

$$\sqrt{\left(x_S(t) - x_L(t)\right)^2 + \left(y_S(t) - y_L(t)\right)^2} \geq d_{\min}$$

$$\forall t \in \left[0, t_n\right] \tag{17}$$

The minimization of t_n (equation 14) under constraints (15-1) to (15-n), (16) and (17) is a mathematical programming problem which presents serious difficulties:

- it is a mixed variables programming problem (the ε_i are binary variables, the angles θ_i are real and the length l_i are real positive variables),
- the different constraints define a feasible space whose reduction to its continuous facet is not convex and which present an increasing complexity with higher orders,
- the minimum separation constraints are dependent on some initial logical conditions.

An exact method, which seems of interest here, once the problem has been discretized, is Dynamic Programming [8]. This method is known to be quite efficient to construct recursively a solution for a separable optimization problem.

OFF LINE GENERATION OF CONVERGENCE TRAJECTORIES

It has been shown above that possible solution trajectories present a large diversity of regular shapes. In general a convergence trajectory will last at least some minutes during which the aircraft can be submitted to wind effects and to inaccuracies of the navigation and guidance systems. Also, during this period, the leading aircraft can be driven by the air traffic control system to modify its flight plan or its guidance references. Then, once the convergence maneuver is started, it will be necessary to restart repeatedly the problem of generation of a new minimum time convergence trajectory. Then it appears interesting to conceive a new system able to perform an on line generation of the current guidance parameters necessary to perform the whole convergence trajectory.

The optimality conditions (10) have not allowed to get a practical numerical solution to an instance (initial relative positions, aircraft speeds and final relative positions) of the minimum time convergence problem. However, they have allowed to get a good insight of the regular structure of optimal convergence trajectories. The retained formulation for problem $P_{n_{\max}}$ does not lead as well to a solution process, which can be effectively run on board.

Nevertheless if the initial conditions are relaxed from problem $P_{n_{\max}}$, it is possible to solve it step by step, starting from the imposed final convergence conditions, using an inverse Dynamical Programming process. Then, it is possible to generate in sequence and in a reverse way the set of triplets $\left(x_0^S, y_0^S, \psi_0^S\right)$ associated to the corresponding n_{max} order feasible optimal convergence trajectories. Then, if the angular range of θ is discretized into N_θ values, such as: $\theta_k = (k-1)\Delta\theta$, k = 1 à N, and if the length of the straight segments is discretized into N_l values, such as: $l_h = (h-1)\Delta l$, h = 1 à N_l, at each stage, the number of generated triplets will be multiplied by $N_l\,(2\,N_\theta\text{-}1)$.

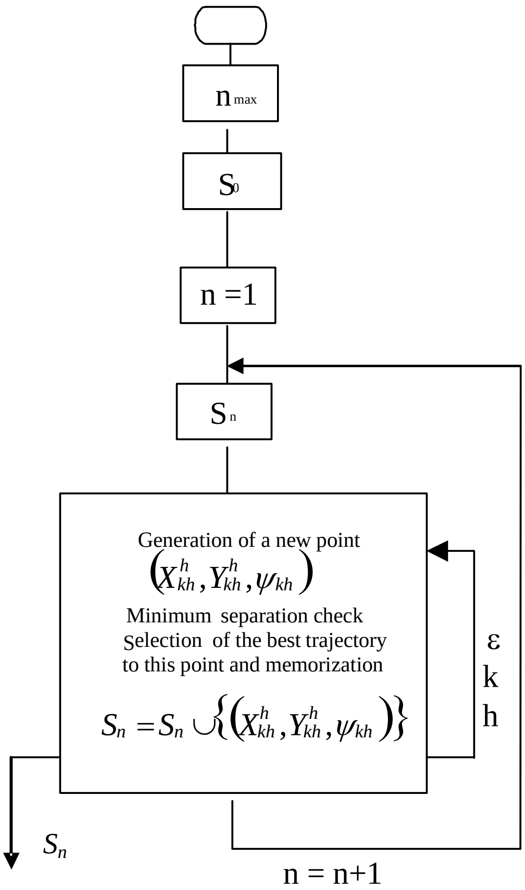

Figure 5: Inverse generation of optimal convergence trajectories sets.

The recursive structure of the algorithm used for generation of the convergence trajectories is displayed in figure 5.

The set of generated trajectories builds a tree whose root is given by (-D, 0, ψ_L). in a reference frame of $R^2X[0,2\pi]$ attached to the estimated final position of the leading aircraft (see figure 6). To each point in this space are then associated the parameters of an optimal regular trajectory of minimal order. This input-output data can be memorized in a data base to give grounds to an on line interpolation by the relative guidance computer of the trailing aircraft.

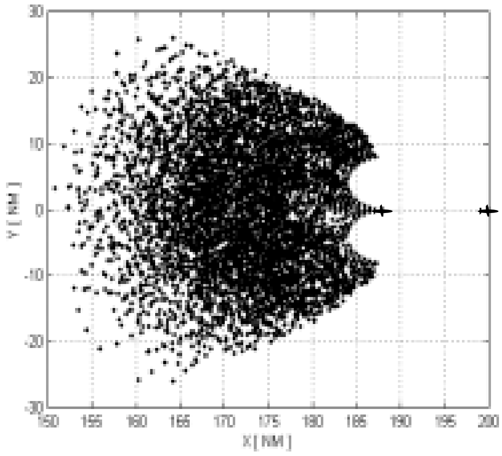

Figure 6: Discretization of the convergence maneuver relative space.

Then at regular time intervals (some seconds of duration), the relative guidance computer of the trailing aircraft must assess its relative position with respect to the leading aircraft. Then it will check in this database which absolute guidance parameters have to be adopted momentarily to goes on efficiently with the convergence maneuver. Since in many situations, no database input is exactly identical with the current situation of the trailing aircraft, an interpolation appears necessary. This can be achieved using a classical feed forward neural network device as it has been proved to be effective in [9] (see figure 7 where x and y are the position, ψ is the heading and ϕ is the bank angle of the trailing aircraft).

This system can provide elements to perform the following advanced functions:

Generation of current guidance parameters for the absolute guidance function of the classical auto pilot of the aircraft, and visualization of the current intended convergence trajectory on the navigation display system [10] (see a possible display in figure 8),

- Communication of intended trajectory information to the air traffic control system in charge of the current traffic sector.

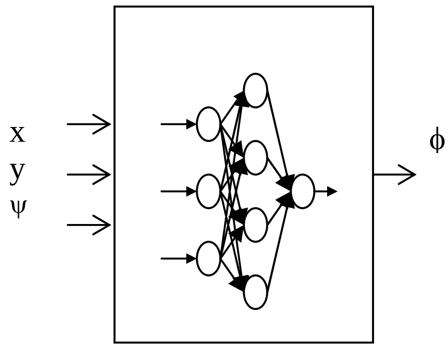

Figure 7: Example of feed forward neural network

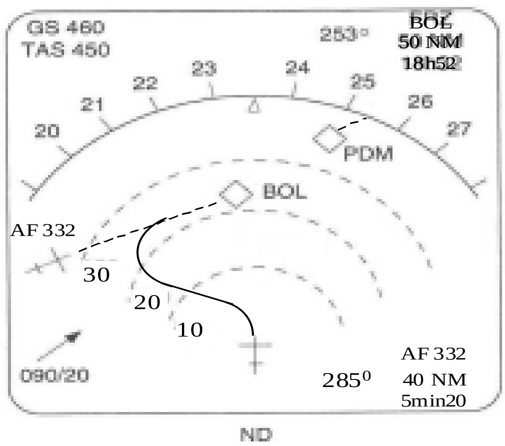

Figure 8: Example of possible convergence trajectory representation on a Navigation Display

In figure 8 the navigation display (ND) of an in trail aircraft is presented. This in trail aircraft is to start a convergence maneuver towards flight AF332, this maneuver is estimated to last 5 minutes and 20 seconds over a distance of 40 nautical miles. Various numerical simulation experiments involving two accurate A300B simulation models have been performed. The new guidance function has been integrated into the simulated autopilot of the trailing aircraft and has provided smooth convergent trajectories. Different case studies have been considered, including some in which the leading aircraft follows a straight route (figure 9) and some in which the leading aircraft performs lateral maneuvers (figure10).

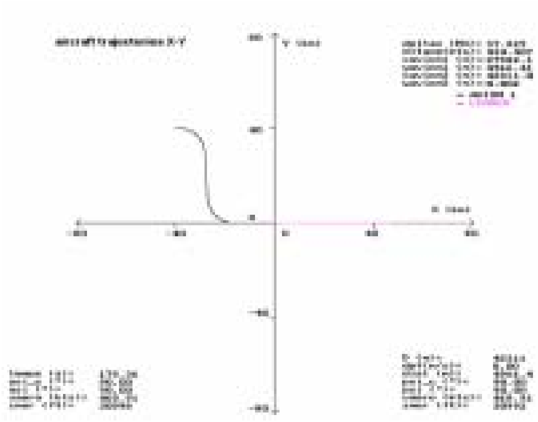

Figure 9: Example of steady convergence maneuver

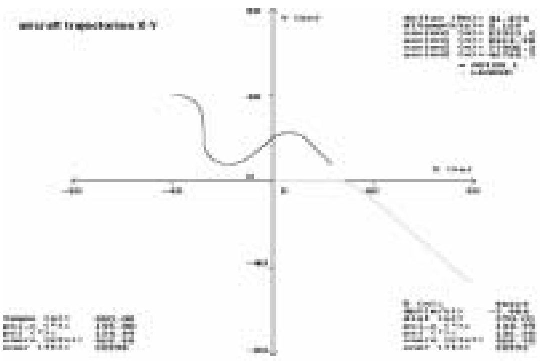

Figure 10: Example of swirling convergence maneuver

CONCLUSION

In this communication the relative guidance of an aircraft with respect to another has been formulated as a minimum time control problem. Optimality conditions have been derived and their analysis has shown that the minimum time convergence trajectories can be characterized by some few geometric parameters. Then a new mathematical programming problem can be formulated from the original optimal control problem to generate sets of regular convergence trajectories. Since the evolution of aircraft is in general subject to perturbations it appeared necessary to provide in real time updated directives to the guidance system of the aircraft. A practical solution strategy, composed of two steps, has been proposed and tested by numerical simulation: First, an off line exhaustive solution approach, based on reverse dynamic programming, has been performed to cover a large set of possible initial relative positions. - Second, a set of feed forward neural networks has been built and trained to associate to any current relative position the corresponding guidance directives along the local optimal regular minimum time trajectory. Then it is up to the auto pilot control laws to invert the flight mechanics equations to determine from these guidance directives, the corresponding input signals to the flight control channels of the in trail aircraft. It appears that the proposed solution is compatible with modern on board guidance systems and may contribute to enhance booth safety and capacity of the air traffic system.

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